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Prediction of Temperature and Velocity distribution in an air conditioned room

Peter V. NIELSEN
DANFOSS A/S
NORDBORG
Denmark

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1. Introduction.

The air conditioning system must meet a number of requirements. Firstly it must remove or supply heat in order to maintain a convenient temperature level and to supply the room with a given amount of fresh air. However, it is not sufficient that the air is supplied in the correct amount with the correct temperature, it must also be distributed with regard to the heat sources in order to obtain a constant temperature level. It is also essential that this distribution is achieved by good mixing with the room air and without causing high velocities.

Fig. 1 illustrates the problem of predicting these amounts. This shows a sectional elevation of a room. The air is supplied from a diffuser at the ceiling and develops into a wall jet running under the ceiling. In this jet it is easy to predict mixing, temperature and velocity profiles, and the diffuser is often so dimensioned that the velocity falls to 2o-25 cm/s when the jet has reached 3/4 of the length of the room.

Away from the ceiling region, however, the simple relation between say velocity and distance from diffuser will disappear - the jet is said to disperse - and other parametres such as the dimensions of the room will have a strong influence on the flow. If a calculation of mixing, temperature and velocity profiles in the occupation zone is wanted, all these parametres must be taken into consideration.

This article presents the outcome of predictions of the flow in an air conditioned room. The method of analysis is based on the solution of the equations of motion and energy transport for the flow - five non-linear partial differential equations - by means of a numerical method. The solutions of this system will provides detailed information on the flow in all regions of the room, including the occupation zone. The solving procedure is described in detail in the reference [1] .

2. Equations.

The equations applied here are derived from the Navier-Stokes equations, the energy equation, and the equation of continuity. We suppose that the air movement in the section of the room under study may be regarded as steady and two-dimensional. This requirement does not exclude the use of diffusers, which at first give a three-dimensional air movement, provided that the main body of air circulation away from the diffusers is steady and two-dimensional.

Although the aforementioned equations represent a complete description of the turbulent flow, it is not practicable to solve these by means of a numerical method. The equations must be supplemented with a turbulence model. The turbulence model used here consists of a transport equation for turbulent kinetic energy k and an equation for the dissipation of turbulent kinetic energi ϵ . These equations are (1) and (2) respectively. Equation (3) shows how the turbulence is described by "turbulent viscosity" μ_t .

$$\frac{\partial}{\partial x_1}\left(k\frac{\partial\psi}{\partial x_2}\right)-\frac{\partial}{\partial x_2}\left(k\frac{\partial\psi}{\partial x_1}\right)=\frac{\partial}{\partial x_1}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_2}\right)$$

$$+\mu_t\left[2\left(\left(\frac{\partial v_1}{\partial x_1}\right)^2+\left(\frac{\partial v_2}{\partial x_2}\right)^2\right)+\left(\frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1}\right)^2\right]-\rho_o\epsilon \qquad (1)$$

$$\frac{\partial}{\partial x_1}\left(\epsilon\frac{\partial\psi}{\partial x_2}\right)-\frac{\partial}{\partial x_2}\left(\epsilon\frac{\partial\psi}{\partial x_1}\right)=\frac{\partial}{\partial x_1}\left(\frac{\mu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\frac{\mu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x_2}\right)$$

$$+c_1\mu_t\left[2\left(\left(\frac{\partial v_1}{\partial x_1}\right)^2+\left(\frac{\partial v_2}{\partial x_2}\right)^2\right)+\left(\frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1}\right)^2\right]\frac{\epsilon}{k}-c_2\rho_o\frac{\epsilon^2}{k} \qquad (2)$$

$$\mu_t = c_\mu \rho_o \frac{k^2}{\epsilon} \qquad (3)$$

ψ is the stream function mentioned later and v_1 and v_2 are mean velocities. The factors σ_k, σ_ϵ, c_1, c_2, and c_μ are various constants. The development of the equations is described more explicitly by Launder et al [2] .

Transport equations are made for the vorticity ω and the stream function ψ , derived from the Navier-Stokes equations.

$$\frac{\partial}{\partial x_1}\left(\omega \frac{\partial \psi}{\partial x_2}\right) - \frac{\partial}{\partial x_2}\left(\omega \frac{\partial \psi}{\partial x_1}\right) = \frac{\partial}{\partial x_1}\left(\frac{\partial \mu_t \omega}{\partial x_1}\right)$$

$$+ \frac{\partial}{\partial x_2}\left(\frac{\partial \mu_t \omega}{\partial x_2}\right) + \rho_o \beta g_2 \frac{\partial T}{\partial x_1} \tag{4}$$

$$\frac{\partial}{\partial x_1}\left(\frac{1}{\rho_o} \frac{\partial \psi}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\frac{1}{\rho_o} \frac{\partial \psi}{\partial x_2}\right) = -\omega \tag{5}$$

ρ_o , β and g_2 represent density, coefficient of expansion and gravitational acceleration, respectively, and T is the temperature.

The vorticity is twice the angular velocity of the air. The stream function describes the vector field (v_1, v_2) by a single scalar quantity ψ . It is a practical variable when describing an air conditioning problem, because lines through constant ψ -values are stream lines, i.e. lines parallel to the mean velocity vector.

The energy equation rounds off the equation system used, and it is here given in the form

$$\frac{\partial}{\partial x_1}\left(T \frac{\partial \psi}{\partial x_2}\right) - \frac{\partial}{\partial x_2}\left(T \frac{\partial \psi}{\partial x_1}\right) = \frac{\partial}{\partial x_1}\left(\frac{\mu_t}{\sigma_h} \frac{\partial T}{\partial x_1}\right)$$

$$+ \frac{\partial}{\partial x_2}\left(\frac{\mu_t}{\sigma_h} \frac{\partial T}{\partial x_2}\right) \tag{6}$$

where σ_h is the turbulent Prandtl number.

The five differential equations (1), (2), (4), (5), and (6), having the five unknown factors ω , ψ , T, k and ϵ , represent a complete description of the air flow, and they form the basis of the numerical solution method.

Fig. 2 shows the dimensions and factors which characterize an air conditioned room. If the equation of continuity, the Navier Stokes equations and the energy equations are rewritten in a dimensionless form it is possible to demonstrate that the flow is completely described by the geometrical relations, the Reynolds number Re and the Archimedes number, Ar.

$$Re = \frac{V_o\, h\, \rho_o}{\mu_o} \qquad (7)$$

$$Ar = \frac{\beta\, g_2\, h\, \Delta T_o}{V_o^2} \qquad (8)$$

The height h at the diffusor is selected as the representative length. The representative velocity is the diffusor velocity V_o and the representative temperature is the temperature difference between inlet and outlet ΔT_o.

The Archimedes number represents a sort of ratio between free convection and forced convection. Using the Archimedes number it is possible to arrive at three characteristic situations:

1. In the case of isothermal ventilation the Archimedes number is Ar = 0. The energy equation (6) and the temperature T do not figure in the problem. The Reynolds number and the geometrical relations characterize the flow.

2. In situations where the temperature differences are so small that forced convection is predominant in relation to free convection, a situation arises where the temperature distribution does not affect the flow. The Archimedes number is relatively small. The Reynolds number and the geometrical relations characterize the flow.

3. In normal situations free convection can be of importance. The Archimedes number has a significant size and this number together with the Reynolds number and the geometrical relations, characterize the flow.

3. Numerical method.

The principle of the numerical method is to replace the differential equations by a number of difference equations which can be solved by a systematic procedure. The section under study is divided up into a number of points. The five differential equations are replaced by five difference equations in each point. If a point number of 21 x 21 is used, it results in an equation system consisting of 21 x 21 x 5 = 22o5 equations with 22o5 unknown factors. This equation system is solved in a computer of the size ~ loo K bytes by a modified Gauss iteration. The examples given in this paper have been produced on an IBM system 37o/145.

The development of the difference equations and the structure of the iterative procedure are described in detail by Gosman et al [3].

4. Results.

In the following, some comparisons will be made between test results from model experiments and numerical predictions. The details of the experiments are discussed in the reference [1].

Fig. 3 shows a vertical velocity profile. It is measured at a distance of 2/3 L from the diffusor - see the marking on fig. 2. This distance is chosen because the velocity profile in this area passes through the centre of the recirculating flow and therefore has components mainly in horizontal directions. Also, the velocity profile is placed in the area where the velocity is maximum at the floor of the model.

By means of a hot-wire anemometer it is possible to measure both the mean velocity v_1 and, in some areas, the instantaneous deviation from the mean velocity v_1' in the direction x_1.

Both measured and predicted velocity profiles are made dimensionless in relation to the respective diffusor velocities V_o. It will be seen that the prediction gives a satisfactory velocity profile over the whole area.

It is also possible to compare the predicted turbulence energy k with the measured turbulent intensity $\sqrt{\overline{v_1'^2}}$ i.e. the flow can be characterized as a free shear layer.

$$\sqrt{\overline{v_1'^2}} \sim \sqrt{k} \qquad (9)$$

Fig. 3 illustrates this comparison. It will be seen that the used turbulence model can reproduce the relatively high turbulence level at the lower floor. This is an important factor since it demonstrates that reasonable consideration is given to the transport phenomena in the turbulent flow.

Fig. 4 shows a comparison between measured and predicted temperature profiles. Temperature distribution is brought about by supplying a constant flow of heat through the floor of the model. The upper curve illustrates the temperature distribution through an area corresponding to the occupation zone in a room, see fig. 2. The lower curve shows the distribution of surface temperature at the floor. The temperature is shown dimensionless on the figure.

The reference [1] shows that the influence of thermal radiation in a model is insignificant. Therefore the prediction does not take into account thermal radiation between the surfaces. It will be seen that measured and predicted temperature profiles largely agree both in respect of air temperature and surface temperature.

Comparisons between experiments and predictions have also been made in a full scale test room. The room has dimensions of 2.8 x 3.4 x 8.3 m with a diffusor of the size h/H = 0.005. These comparisons are described in detail in references [1] and [4] .

5. Conclusion.

Comparison with test results shows that the suggested prediction method is suitable for investigation of the flow in an air conditioned room when the flow is steady and two-dimensional in the section under study.

The prediction method gives the required information for the evaluation of thermal comfort, i.e.: air velocity, air temperature, surface temperature, velocity and temperature gradients, and turbulence intensity.

The prediction method can be extended to other physical factors. Transport equations of the same type as (6) can be developed for air-water vapour mixture and for concentration of particles in air. The solution of these equations will be of interest in connection with cold store and "clean room" technology respectively.

6. References.

[1] Nielsen, P.V., Ph.D.-Thesis, In Print.

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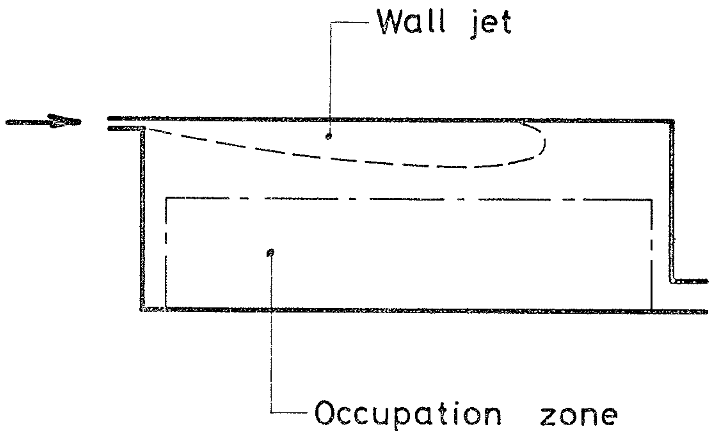

Fig. 1. The section under study in an air conditioned room.

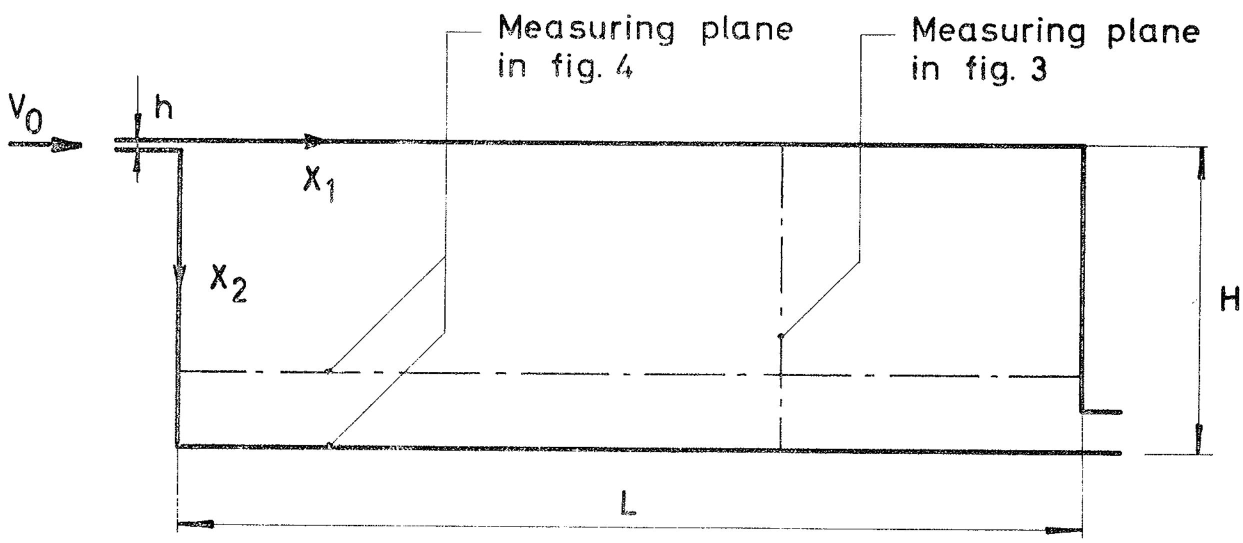

Fig. 2. Dimensions which characterize an air conditioned room. Marking of the measuring plane in fig. 3 and fig. 4.

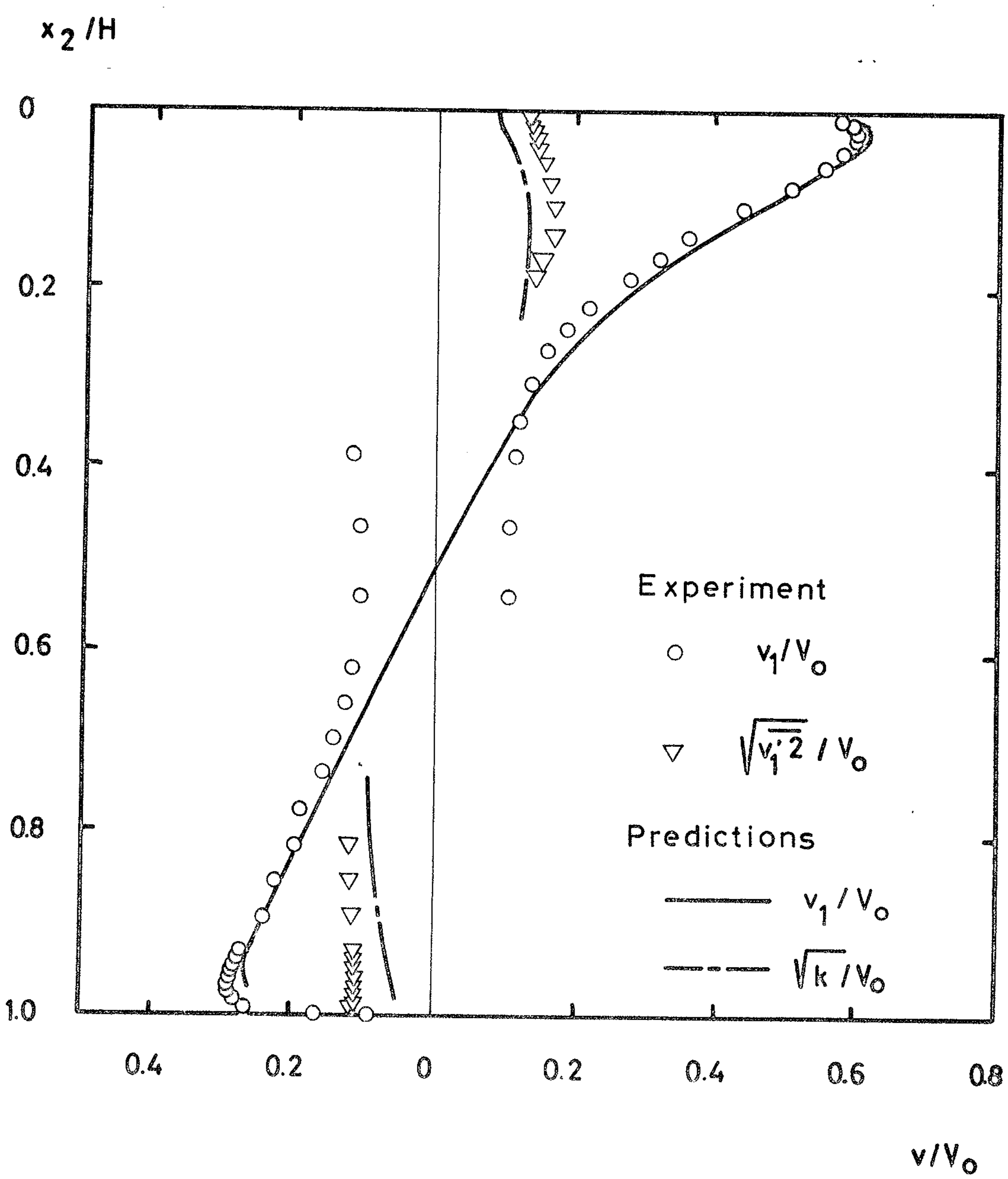

Fig. 3. Comparison between predicted and measured velocity and intensity profile. h/H = 0.056, L/H = 3.0, Ar = 0 and Re = 7100. From reference [1] .

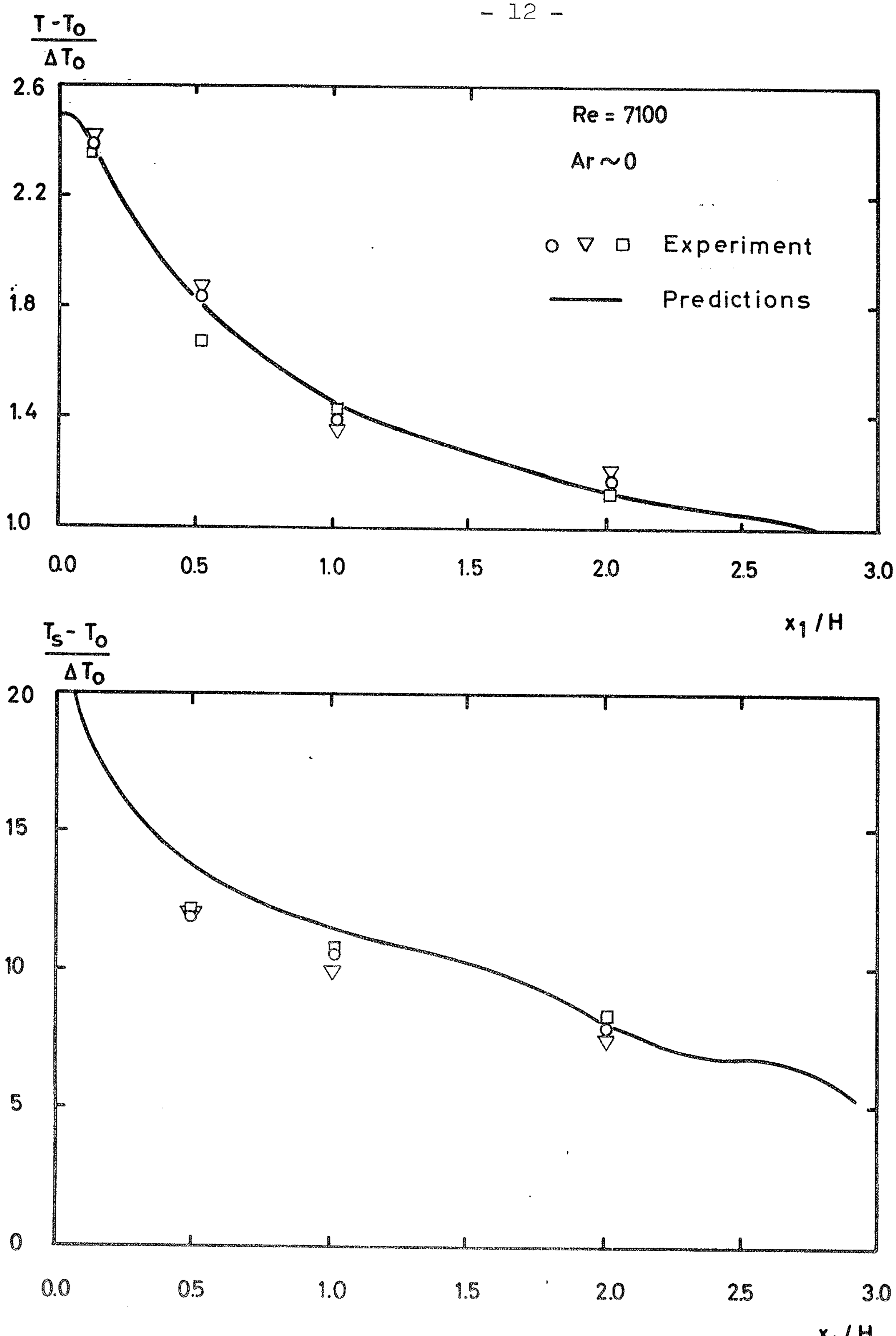

Fig. 4. Comparison between predicted and measured temperature profile. Air temperature T and surface temperature T_s. $h/H = 0.056$ and $L/H = 3.0$. From reference [1].