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A Modified Low-Reynolds-Number *k*-ω Model for Recirculating Flows

A modified form of Wilcox's low-Reynolds-number k-ω model (Wilcox, 1994) is proposed for predicting recirculating flows. The turbulent diffusion for the specific dissipation rate, ω, is modeled with two parts: a second-order diffusion term and a first-order cross-diffusion term. The model constants are re-established. The damping functions are redevised, which reproduce correct near-wall asymptotic behaviors, and retain the mechanism describing transition as in the original model. The new model is applied to channel flow, backward-facing step flow with a large expansion ratio (H/h = 6), and recirculating flow in a ventilation enclosure. The predictions are considerably improved.

1 Introduction

Turbulence modeling is one of the most important aspects in numerical simulations of fluid flow and heat transfer. In conjunction with empirical wall functions, the conventional k-ϵ model (Launder and Spalding, 1974) has been widely used in engineering practice, and has turned out a success in many applications. Nevertheless, some problems exist when using the wall-function method (Patel et al., 1984). The lack of universality of the wall functions has been frequently criticized. Extensive research has thus been made to develop near-wall low-Reynolds-number (LRN) corrections.

Most of LRN models have been developed based on the k-ϵ model. A major deficiency with the k-ϵ-based LRN models is the uncertainty of specifying ϵ at the wall. There are other deficiencies with some LRN k-ϵ models (Peng et al., 1996a). The damping functions used in LRN models usually rely on wall-proximity-dependent variables (e.g., $y^+ = u_\tau y/\nu$ and $R_y = k^{1/2}y/\nu$ etc.). This gives rise to complications in numerics when solving for internal flows with non-planar wall geometries. Further, Savill (1995) has indicated that LRN models that use damping functions dependent on the turbulent Reynolds number, R_t, are more appropriate for predicting low-Reynolds-number transitions than those that only introduce a dependence on wall proximity. In addition, for some turbulent recirculating flows of engineering interest, e.g., low-velocity displacement ventilation flows, where laminar and transitional phenomena exist locally not only in near-wall regions but also in regions remote from the walls, using an LRN k-ϵ model often turns out unrealistic laminar solutions (Davidson, 1989).

In recent years, some new LRN two-equation models have been proposed as alternatives to the LRN k-ϵ models, e.g., the LRN k-τ model by Speziale et al. (1992). Wilcox has developed a standard two-equation k-ω model (Wilcox, 1988) and its LRN variant (Wilcox, 1994). The standard k-ω model has been validated for predicting boundary layer and free shear flows (Wilcox, 1988). Combining the standard k-ω model with the k-ϵ model, Menter (1994) developed two new models, and obtained predictions improved for adverse pressure gradient flows. Patel and Yoon (1995) used the standard k-ω model to solve separated flows over rough surfaces, and reported good accuracy. Abid et al. (1995) used the k-ω model in combination with an explicit algebraic stress model for recirculating flows, and obtained results in good agreement with experiments. Larsson (1996) applied the k-ω model to predictions of turbine blade heat transfer, and concluded that the k-ω model performs as well as or better than the k-ϵ model. Some other recent applications with the standard k-ω model can also be found, see e.g., Liu and Zheng (1994) and Huang and Bradshaw (1995). With both the wall-function method and the extended-to-wall method (integrating the model directly towards the wall surface), Peng et al. (1996b) recently applied the standard k-ω model to recirculating flows. It was found that this model overpredicted the reattachment length for the flow behind a backward-facing step with a large expansion ratio ($H/h = 6$).

With the LRN k-ω model, satisfactory results have been reported for simulating transitional boundary layer flows by means of the concept of numerical roughness strip (Wilcox, 1994). One of the attractive features of Wilcox's LRN model is that it uses damping functions that depend only on the turbulent Reynolds number, R_t. It is therefore convenient to apply this model to internal flows with complex geometries. Moreover, the k-ω model possesses a nontrivial laminar solution for ω as $k \to 0$. It can thus be expected to be able to capture flow characteristics for e.g. low-velocity displacement ventilation flows of which an LRN k-ϵ model fails to handle. However, the LRN k-ω model, as its standard form, yields significant inaccuracy in predictions for the flow over a large backward-facing step. Furthermore, this model does not reproduce correct asymptotic behavior for $-\overline{u'v'}$, with $-\overline{u'v'} \propto y^4$ as $y \to 0$. It has been argued that the correct wall-limiting condition for $-\overline{u'v'}$, as well as for ϵ, contributes to the improvement of predictions of by-pass transitions (Savill, 1995).

This paper presents an improved form of the LRN k-ω model. A turbulent cross-diffusion term is added to the modeled ω-equation, in analogy to its viscous counterpart in the exact transport equation. The model constants are re-established. New R_t-dependent damping functions are devised to make the model asymptotically consistent as the wall is approached. In addition, the mechanism for simulating boundary layer transitions is pre-

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