A Discussion of Low Reynolds Number Flow for the Two-Dimensional Benchmark Test Case

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Introduction.

The use of CFD in ventilation research has arrived to a high level, but there are some conditions in the general CFD procedure which do not apply to all situations in the ventilation research. An example of this is the turbulence models in Reynolds-averaged Navier-Stokes equations, i.e. (RANS) equations.

The flow in a ventilated room is generally assumed to be a fully developed turbulent flow, and this flow can be handled by most turbulence models. But in some areas of the room, including the occupied zone, a low Reynolds number flow can exist at a low room air supply velocity. Figure 1 shows measurements of the maximum velocity in the occupied zone of a room with mixing ventilation from a wall-mounted diffuser versus the air change rate. The flow is isothermal (Nielsen, 1992). Similarity principles state that any velocity, e.g. the maximum velocity in the occupied zone, is a linear function of the air change rate (or the supply velocity) when the flow is a fully developed isothermal turbulent flow. In Figure 1 this is the case for velocities larger than 0.25 m/s, but the figure indicates that the flow in the occupied zone is a low Reynolds number flow for velocities below 0.25 m/s (dotted line). The conventional turbulence models cannot accurately capture the flow in this low Reynolds number regime. On the other hand, an air change rate up to 5 covers most of the practical cases in air conditioning.

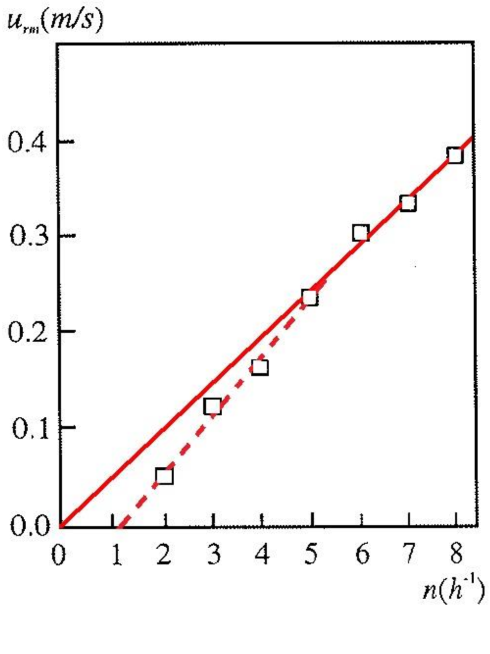

Figure 1. Measurements of maximum velocity in the occupied zone of a room versus air change rate (supply velocity) in the case of isothermal mixing ventilation. Proportionality between supply velocity and maximum velocity in the room indicates a fully developed turbulent flow in the occupied zone for a supply velocity larger than 0.25 m/s, and a low Reynolds number flow for lower velocities.

Model and equation systems

The problems with a low Reynolds number flow will be addressed in the following IEA 2D test case.

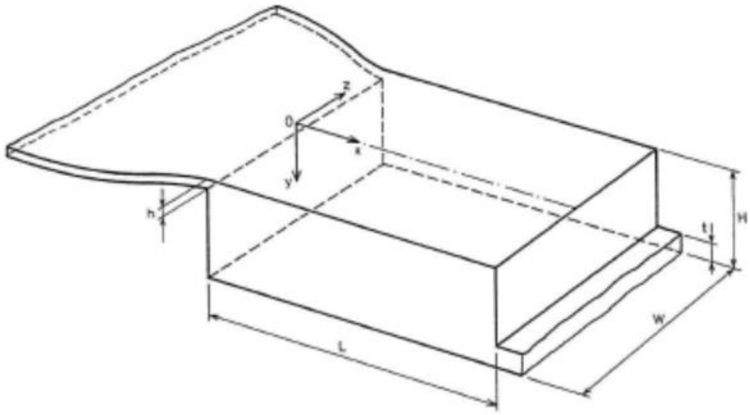

Figure 2. The two-dimensional benchmark test, also called the "IEA 2D test case".

The geometry of this benchmark model is described as:

H = 3m, L = 9m, h = 0.168m, t = 0.48m

The air is supplied from the top slot in the model and the return air flow is through the bottom slot on the opposite position.

The boundary conditions for the following CFD predictions are given as:

Inlet velocity: u_o

Turbulent kinetic energy: $k_0 = 1.5(0.04 \bullet u_0)^2$

Dissipation: $\varepsilon_0 = \frac{{k_0}^{1.5}}{l_0}$ where $l_0 = h/10$

The Reynolds number is defined as $Re = \frac{h \bullet u_0}{\nu}$

According to previous studies, the model has a two-dimensional steady state flow (Olmedo et al. 2010), and there are more than 50 papers related to this model including both experimental and numerical studies made during recent years.

Mathematical description

The flow will be studied in the isothermal case and is fully described by the two-dimensional Reynolds-Averaged Navier-Stokes equations (RANS) without the energy equation.

Mass conservation equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

Momentum conservation equation (Navier-Stokes equations)

$$\frac{\partial(\rho u)}{\partial t} + u\frac{\partial(\rho u)}{\partial x} + v\frac{\partial(\rho u)}{\partial y} = \frac{\partial}{\partial x}\left(\mu_{eff}\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu_{eff}\frac{\partial u}{\partial y}\right) - \frac{\partial p}{\partial x}$$

$$\frac{\partial(\rho v)}{\partial t} + u\frac{\partial(\rho v)}{\partial x} + v\frac{\partial(\rho v)}{\partial y} = \frac{\partial}{\partial x}\left(\mu_{eff}\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu_{eff}\frac{\partial v}{\partial y}\right) - \frac{\partial p}{\partial y}$$

The equation system describes laminar flow when μ_{eff} is equal to the physical viscosity μ, and it describes the averaged values in fully developed turbulent flow when μ_{eff} is equal to $\mu + \mu_t$, where the distribution of the turbulent viscosity μ_t is given from the k-ε equations.

A laminar flow prediction can thus be made by the above equations as well as a prediction with fully developed turbulent flow.

The k-ε turbulence model is only valid for fully turbulent flow and it is therefore not possible to make predictions in the regime between the laminar and the fully developed turbulent flow. A Low-Reynolds k-ε model (LRN) sheds some light on this area. The LRN model is developed for wall boundary layer and is basically a k-ε model with variable “constants” (c_μ and c_2) adjusted by a local Reynolds number R_t. Launder and Sharma (1978) introduce a version of an LRN model with the variable “constants” f_μ and f_2 given as functions of the local turbulent Reynolds number R_t

$$f_\mu = \exp(-3.4/(1 + R_t/50)^2)$$

$$f_2 = 1- 0.3 \exp(-R_t^2)$$

A high turbulence k-ε model, which is used in this paper, may be considered a special version of an LRN model with f_μ and f_2 equal to 1.0. Those conditions are fulfilled in practice when R_t exceeds 400 everywhere in the flow domain, as seen from the above equations. Nielsen (1974 and 1995).

The local turbulent Reynolds number can be expressed by

$$R_t = \mu_t/(c_\mu f_\mu \mu)$$

Fully developed turbulent flow ($R_t > 400$ and $c_\mu = 0.09$) corresponds to the ratio

$$\mu_t/\mu > 40$$

The value of this ratio made it possible to express the quality of a CFD prediction with a *k-ε* turbulence model. An LRN model is identical to a *k-ε* model when the ratio is larger than 40, and the *k-ε* model is therefore a valid model in this case. When the ratio is smaller than 40, the local turbulent Reynolds has such a level that the *k-ε* model is insufficient to describe the flow with the level of low Reynolds number turbulence which exist in the area. The following chapter addresses the possibilities of using the ratio μ_t/μ for quality control of a prediction.

Predictions at low Reynolds numbers based on a *k-ε* model.

Figure 3 shows the predictions at Reynolds numbers from 5000 down to 500.

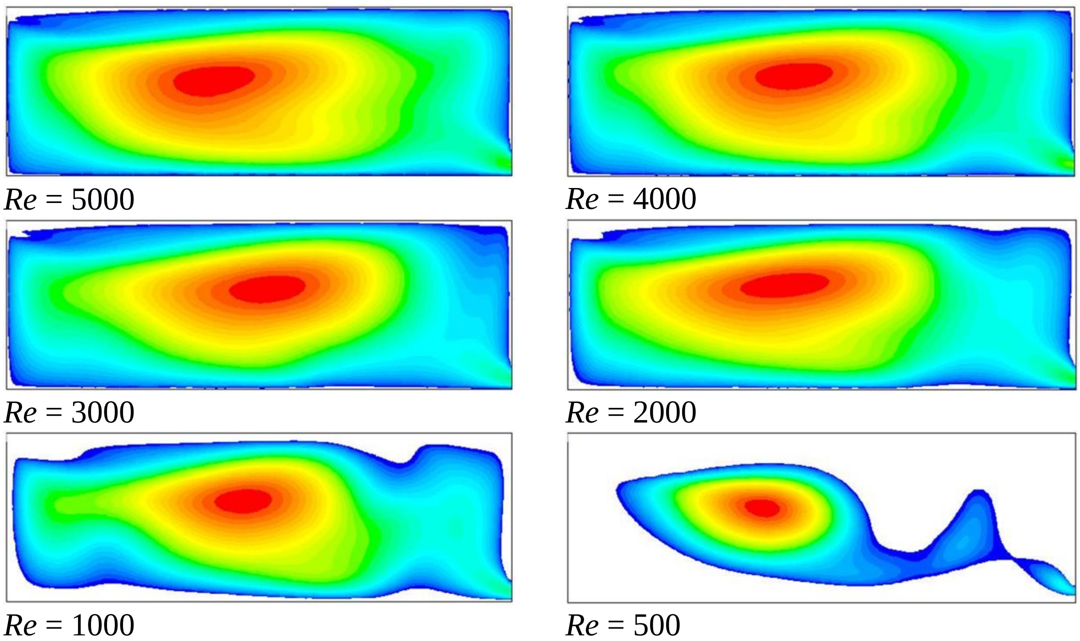

Figure 3. Prediction of the flow in the IEA 2D test case. Areas with a value of the turbulent viscosity lower than 40 μ are indicated as a white area.

The predictions show that, even for the Reynolds number of 5000, there will be areas with a turbulent viscosity lower than 40 μ. This is particularly the case for the inlet opening, because the given inlet conditions correspond to a turbulent intensity of 4 %, which is far from the turbulent flow. Le Dréau et al. (2012) have shown that μ_t/μ is about 5 to 10 in the inlet area. This low level of turbulence is maintained in the constant velocity core in front of the opening because there will not be any production of turbulent kinetic energy in this area. The wall jet below the ceiling will immediately obtain a high turbulent level due the production in the velocity gradients. This turbulent level will be transported around in the room in addition to further production.

The predictions for *Re* = 4000 show that μ_t/μ is larger than 40 in most of the domain, and it is also similar to the predictions for *Re* = 5000. Therefor it can be considered to be a fully developed flow.

The predictions for Re = 500 and 1000 shows large areas where μ_t/μ is smaller than 40 and it is only in the center of the recirculating flow that a high turbulence is predicted. Those predictions will be false because they are connected to an area with an insufficient turbulence model. The *k-ε* cannot be used at those low Reynolds numbers.

Predictions with the k-ε turbulent model in situations close to laminar flow.

It is often expressed that the k-ε model can give a sufficient solution at very low velocities (laminar flow regime). This situation will be addressed in this section.

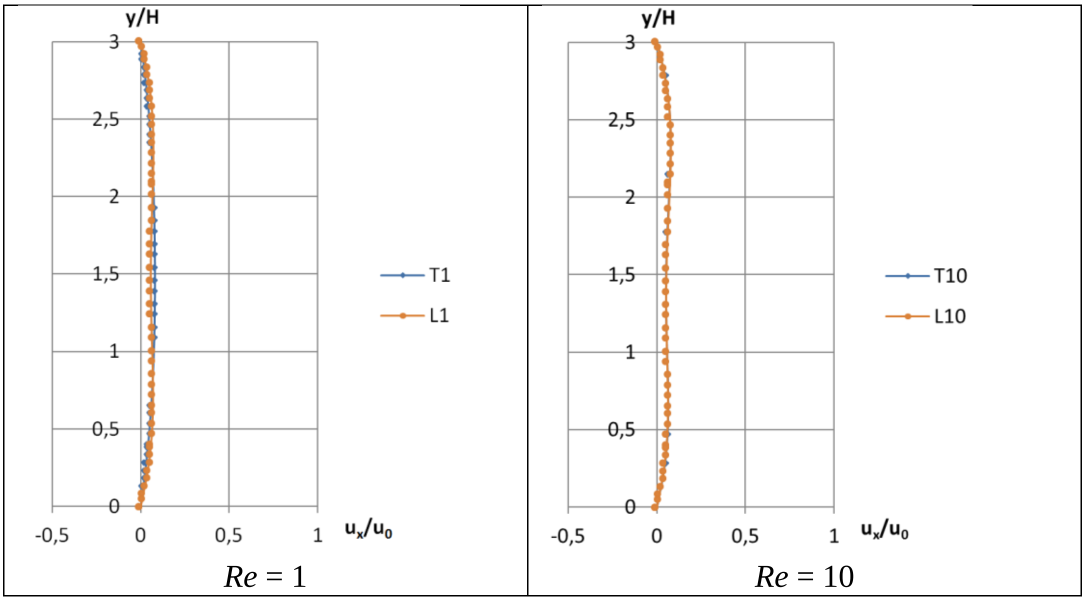

Figure 4. Velocity distribution at x/H = 2.0 predicted by RANS equations and a k-ε model as well as by the laminar equations.

Figure 4 shows that the velocity distribution found by the RANS equations and a k-ε model are identical to the solution found by the laminar equations at the extreme low velocities (*Re* = 1, 10). This effect can be explained if we look into the distribution of μ_t and $\mu + \mu_t$. Figure 5 shows that the predicted turbulent viscosity μ_t from the k-ε model is close to 0.0 for Re = 1 and 10. The molecular viscosity μ will be large compared to μ_t and the RANS equations will be similar to the equations for laminar flow and will therefore predict laminar flow.

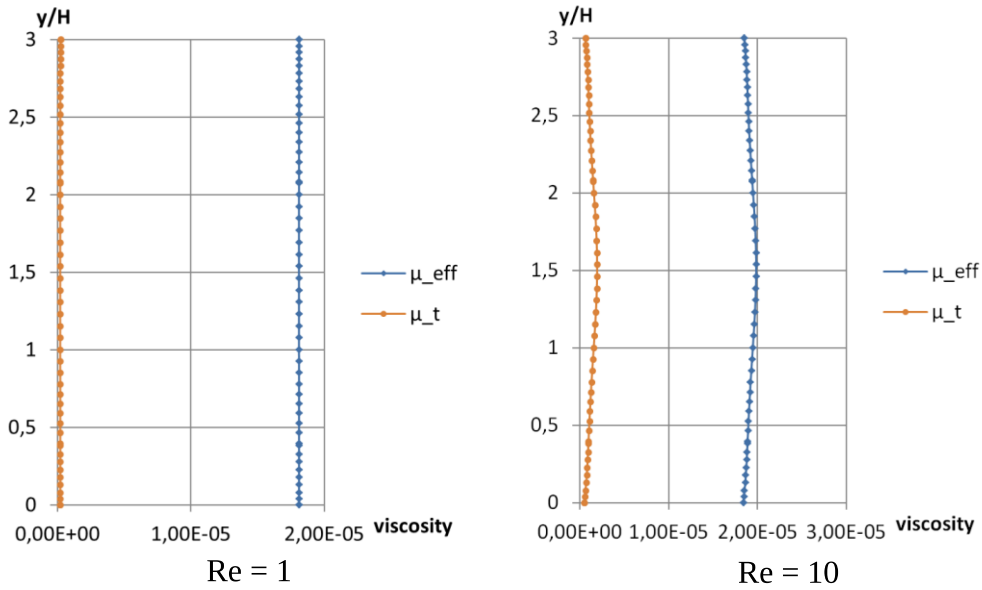

Figure 5. Distribution of μ_t and μ_{eff} = (μ + μ_t) for Re = 1 and 10, at the position x/H = 2.0.

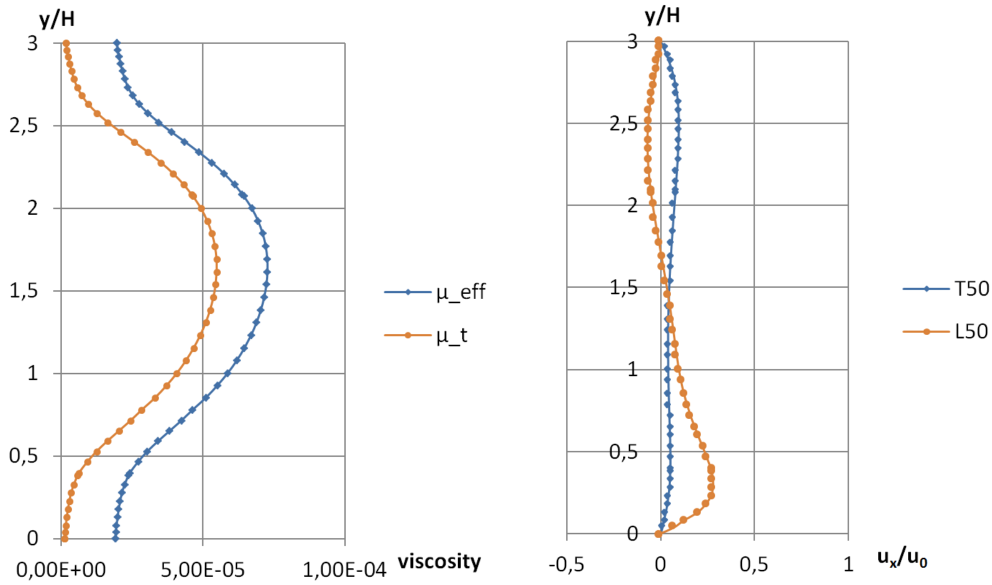

Figure 6. Distribution of μ_t and μ_{eff} = (μ + μ_t) for Re = 50, as well as velocity distribution at the position x/H = 2.0.

The prediction of μ_t in the RANS equations will amplify with an increasing Reynolds number and will be about 2 to 4 times larger than the molecular viscosity μ (but of course far from 40), see figure 6. The velocity distribution shows two very different results. The RANS solution is surely wrong and only measurements can determine the real solution.

Conclusion

In the past couple of years, CFD prediction has played an important role in ventilation research and more and more people use this tool. The turbulence model selected in the CFD simulation becomes particular important, and in this paper it is shown that the standard k-ε model is not valid for all cases although the k-ε model is widely used in ventilation research in CFD prediction.

The inability of the k-ε model in the low velocity regime, which exists in ventilation research, has been demonstrated in the IEA Annex 20 2D test case. When the inlet Reynolds number is larger than 4000, the prediction in CFD code with the standard k-ε model reflects the turbulent flow in the real flow. When the inlet Reynolds number is less than 10, the prediction in CFD code with the standard k-ε model is similar to the laminar model, but when the inlet Reynolds number is from 10 to 4000, it is most likely impossible to make predictions with the standard k-ε model.

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